Flue Gas flow dynamics & deposition of Particulate Matter inside an Air Sampler Thimble: A Suggestive study

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Abstract:

Flue gas, generated by combustion of fuel particularly solid fuel is usually rich with Particulate Matter (PM) which in the major pollutant for air pollution issue. The Particulate Matter content may be generated by (2) Unburned Carbon particle (b) Ash particle (e) condensed gas vapour, particularly if existing below dew point. The flue gas is sampled using a Particulate Matter Collector thimble placed inside a Sampling probe tube with a Collector Nozzle attached to a S shaped bent section at the tip of the Probe tube. The probe arrangement is inserted through a Sampling hole into a stack and subjected to negative suction pressure generating an induced flue under iso-kinetic condition through the Thimble by an external Vacuum Pump. The Thimble serves as a screen allowing only flue gas through its pores while obstructing and collecting the Particulate Matter over its internal surface. The following mathematical model investigates the flue gas dynamics through the Thimble and corresponding various forces generated that may contribute for Particulate Matter arrest & deposition inside the thimble.

The Thimble is basically a cylinder with one end sealed as a hemisphere, the other end open. The surface of the Thimble is serrated & spread with micro-pores having characteristic temperature & acid resistance.

Keywords

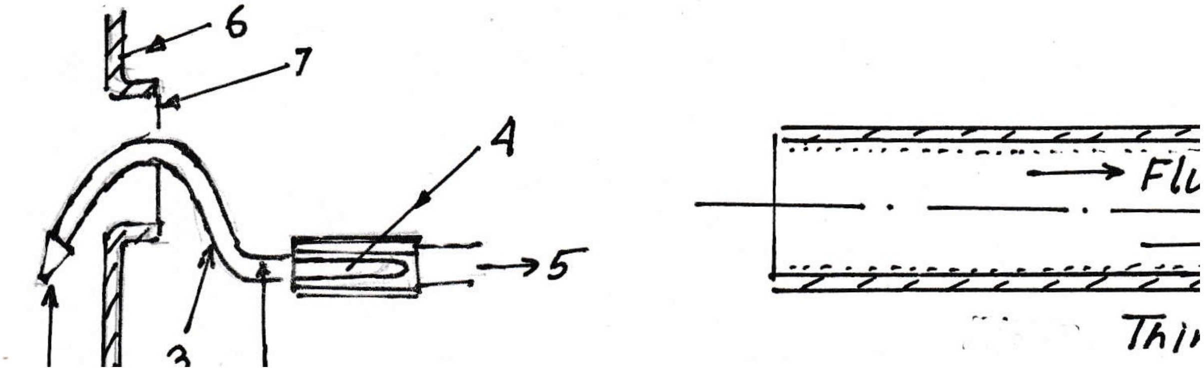
Particulate Matter
Bernoulli’s Equation
Navier Stokes equation
Laminar flow
Turbulent flow
Boundary Layer
Impaction
Interception
Attraction
Diffusion.

Introduction

Particulate Matter collector thimbles are used for Sampling flue gas flow being dissipated through stacks by mounting a Thimble in a sampling probe mechanism with a nozzle attached to a bend at the top of the Probe tube. The entire mechanism is a hollow suction arrangement to draw in a part of stack bound flue gas by using a vacuum pump, under Iso-kinetic condition. The thimble in marked for its initial weight under desiccated condition & also marked for its final weight after the sampling process is complete. The thimble collects the flue gas borne particles through any combination of four processes that may include Mechanical force dominated processes like interception, impaction, diffusion or Electrical force dominated process

Aim & Objective:

The paper aims to provide a fundamental study of the various forces generated by the fluid dynamics inside the thimble for generating the Particulate Matter arrest & deposition over the thimble. The study is expressed in an open mathematical model with further scopes for improvementas indicated implicitly by the broad conclusions in both theoretical & experimental senses.



1.Probe Rod 2. Nozzle 3. Bend 4. Thimble 5. Pump 6. Stack 7. Sampling Hole

Fig : Thimble &Pitot Tube

Mathematical Model of the phenomenon involving momentum, energy & force distribution for Particulate Matter deposition inside the thimble

The thimble for collection of Particulate Matter during fan air sampling motion acts according to a principle similar to collection 0f dust by Bags in a Bagfilter device. Accordingly, the operation is guided by four parallel collection principles determined by **four categories of forces -**

(1) **Impaction force F1**or mechanical adhesion to the thimble surface by loss of kinetic energy & corresponding penetration work generation by Particulate Matter in a forced Convection flow by creating momentum loss . The fundamental governing equation is $\rho V 4/3 \pi r^3$

wherer = radius of the particle,

V = velocity the particle

ρ = density of the particle

Velocity of the particle may be inferred by Maxwell Boltzmann Velocity Distribution Function which is represented by

$$Vf(v)d^3 = (m/2\pi KT)^{3/2} e^{-mv^{2/2KT}} d^3v$$

Where
v= velocity

d^3v = velocity space

m =Particulate Matter mass

K =Boltzman's function

T = temperature

(2) **Interception force (F2)** or mechanical adhesion to the thimble surface by obstruction which may be considered a cognate phenomenon of impaction .

The Energy distribution leading to the field potential distribution in the system which in turn determines the Momentum & Force distribution is provided by the Bernoulli's Equation for an Open Thermodynamic system as the given case is:

$(H + v^2 /2g + z)m + W=Q$

H = enthalpy

V= Velocity

Z= Potential Head

$m = m_f + m_p$ = mass of flue gas (Control Volume) + mass of Particulate Matter

W = Work done =0 for this case

Q = Thermal input from combustion in the process

So, Total fluid Energy input generated for the process.

$= Q + \Delta P$

where ΔP = Pressure differential of the stack

$= \Delta P_1 + \Delta P_2$

Where ΔP_1 =induced pressure head by ID fan
ΔP_2 = natural draught generated by pressure head.

Assuming the flue gas flow adiabatic & Compressible

$P= C_1'g^K (K \rho^{(k-2)} / K-2) + C = f(\rho)$

Where k= adiabatic exponent.C, C' = constant

Assuming W=0 Z=0

$(u + PV + v^2 /2g + z) m +W=Q$ translates to

$u + (C_1'g^K (K \rho^{(k-2)} / K-2) + C) V + v^2/2g + z =Q'$

which in the original equation of the open thermodynamic system .

Assuming Z=0 The energy equation translate into Navier Stokes Equation which ultimately results intoImpaction force (F1) Interception force (F2) for the Particulate Matter by Momentum transfer throughliquid Solid in interface (Flue Gas & Solid interface) as per the mentioned logic of field potential difference as

$PDU/Dt = -\nabla p + \nabla.\Gamma + \rho g$

Assuming $P= f(H, v^2 /2g)$

$PDU/Dt = -\nabla f(H, v^2 /2g) + \nabla.\Gamma + \rho g$

What might be of further interest may be exact nature of viscosity as a functionof temperature. It is assumed that the flue gas has characteristics of Newtonian fluid flue.0The Pressure differential ΔP in the NavierStoke’s equation is one of the causes of impaction force that generates deposition directly or indirectly.

(3) **Attraction force (F3**)which may be due to electrostatic attraction which may be generated by ionisation of the Particulate Matter in an acidic or alkaline environment and subsequent electrostatic charging by induction of the thimble surface by deposition. However the Particulate Matter deposition will generate a repulsion layer similar to Zeta potential unless the induced charge have an alternating nature.

The Attraction force in expressed by as

$$\mathbf{F = K^* Mm/d^2}$$

Where K = Permittivity of the free space

M = deposited mass in the thimble Surface

m= Particulate Matter mass

d = distance between Thimble Surface &Particulate Matter

(4) **Diffusion force (F4)** governed by Fick's law (1st & 2nd) . This fourth force will be a functions of concentrationexpressed as

$J=-D\nabla\psi$ ….(1)

$\partial \psi/\partial t=\nabla\bullet(D\nabla\psi)$….(2)

where J = diffusion flux

∇ = Del operator

ψ = concentration

D=Diffusion Coefficient

The diffusion force generation in expressed by

$J=(D/RT)\ \nabla f$

where R- Universal gas constant

T-Temperature

f -frugality / population of Particulate Matterof species / diameter

However we may consider diffusion force if the corresponding Peclet Number Pe-> $LU/D \gg 1$

where L= characteristic length

U= fluid velocity

D=Diffusion Coefficient

Self repulsion between charged Particulate Matter in the flue gas may further contribute to deposition force distribution.

The total force on aParticulate Matter element is expressed

$F_1 = F_{1x}\, i + F_{1y}\, j + F_{1z}\, k$

$F_2 = F_{2x}\, i + F_{2y}\, j + F_{2z}\, k$

$F_3 = F_{3x}\, i + F_{3y}\, j + F_{3z}\, k$

$F_4 = F_{4x}\, i + F_{4y}\, j + F_{4z}\, k$

which can be represented as the matrix

$$\sum F = \begin{vmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{vmatrix} = \begin{vmatrix} F_{1x} & F_{1y} & F_{1z} \\ F_{2x} & F_{2y} & F_{2z} \\ F_{3x} & F_{3y} & F_{3z} \\ F_{4x} & F_{4y} & F_{4z} \end{vmatrix}$$

$\sum F$ is thus the total settling force for deposition over thimble surface

The arresting forces for Particulate Matter may also include(1) the Caking & Adhering properties of the Particulate Matter which is a function of temperature or formation of weekly adhering Chars (2) the adhering adsorptive properties which is expressed by the Adsorptive potential which is basically change in free energy of adsorption which basically represents the work done for transfer of the adsorped molecule

$G_{ads} = RTln (P_u / P)$

where

G=Adsorptive potential

P_u= Vapour Pressure / Dalton's Partial Pressure

In addition to the Four (04) mentioned major forces & the Bernoulli's equation governing energy distribution , **additional forces that may contribute for the Force** , Energy , Momentum distribution in the discussed phenomenon leading to Particulate Matter deposition are related to the flow of the Particulate Matteras a part of the mixed Binary phase Fluid flow.

(1) The drag force/ resistance force $F = C_D (V^2 /2g)$

Where $C_D$is the Drag coefficient which is a function of particular shape factor

(2)Buoyant / upward body force $F = \rho_{fg} V_p g$

ρ_{fg}– Density of fluid / flue gas replaced V_p– Particulate Matter volume g – gravitational const

(3)The Lift force/ upward force $F = C_L (V^2 /2g)$

Where C_L Lift coefficient as a function of particular shape factor

(4)Gravity force / downward body force $F=\rho V g$

ρ – Density of particle V – particle volume g – gravitational const

(5) Thrust force / forwarding force F = Kinetic head + Pressure head

The characteristic Cunningham Correction factor for correcting this drag force subjected to knudsen number (K_n)

$K_n=\lambda/L$ λ- mean free path for Particulate Matter flow

As a parallel expression $K_n= K_B T/ (\sqrt{2}\,\pi\, d^2) P L$where

K_B= Boltzmann Constant

T =Thermodynamic temperature

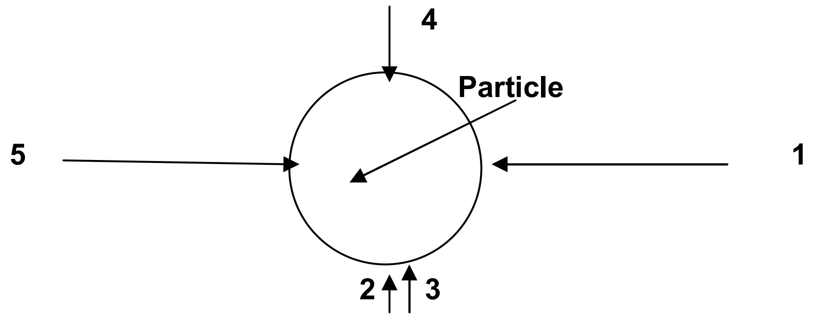
D = Hard shell diameter of particle

P = total pressure

L= Characteristic length of the system

The Cunningham Correction factor being expressed as

$$C= 1+ 2\lambda/d(A_1 +A_2 e^{-A3\, d/\lambda})= f(\lambda, d)$$



1. The drag force/ resistance force **2.** Buoyant / upward body force **3.** The Lift force/ upward force **4.** Gravity force / downward body force **5.** Thrust force / forwarding force

Aerodynamic forces on a floating Particulate Matter

Few special cases / factors may be notedwhich may contribute further interestingly for local &exact distribution of momentum & force resulting in a more precise comprehension for the generation & nature of main impaction forces for Particulate Matter deposition inside the Thimble . These phenomena may be studied further along with empirical studies.

1. The precise nature of momentum transfer is occurring from fluid (represented by Bernoulli's equation) to Particulate Matter as a Binary stage fluid flow.

2.The momentum distribution & the force distribution becomes more noteworthy if the Particulate Matter laden flow becomes rotational at high temperature generating cross sectional velocity gradient demanding rotational Kinetic energy ($\frac{1}{2} m\omega^2$) distribution study.

3.Distribution of momentum in case there in any slip velocity between the solid & fluid flow. Such cases may be assumed as cases of zone based Couette flow where the Particulate Matter rotate amidst fluid flow or move irrotationally wrto fluid flow.

4.Energy & Consequent momentum / force distribution will also be contributed by Viscid fluid flow (flue gas at high temperature) which will result in Couette flow around the gas laden Particulate Matter and various aerodynamic Stresses like Skin friction both at the thimble surface and over individual Particulate Matter. Such forces will also contribute to dissipative work & energy distribution further & momentum distribution between gas (flue gas) & Solid (Particulate Matter). We are disregarding here (1) the possibility of any extended phenomenon related to high Mach number that may generate any separation of Boundary layer for the Particulate Matter elements (2)any generation of vortex path akin to Karman Vortex pathway . The mutual collisions between the Particulate Matter resulting due to momentum transfer from fluid may result in adhesion between particles thus generating some massive Particulate Matter for easy deposition. The extend of such adhesion will depend on primarily density of Particulate Matter in flue gas & temperature.

5.The flue gas flow suffers head loss & from surface roughness of the thimbles which may be neglected assuming characteristic Reynolds number (Re) less than 2100 for the studied event.. Since the flow is assumed Laminar, the overall fluid head loss flowing through the Thimble may be assumed as according to Darcy Weisbach equation

$\Delta P(H_L) = f\,(L/D)(V^2/2g)$

ΔP = Pressure loss

H_L= Head loss

f =Darcy friction factor

L = pipe length

D= hydraulic diameter

V= fluid flow average velocity

g=Gravitational Constant

While contribution of roughness in the thimble surface may be approximated using any suitable formula for geometrically similar pipe flow acknowledging the roughness as ἑ - roughness dimension, d- pipe diameter.

6. If the fluid is unsteady / non uniform

7. Formation of vortex (∇ X u= ω) beyond the Particulate Matter between Particulate Matter& fluid flow & shedding of the of vortices,

8.Growth of the Particulate Matter deposition on the thimble surface which is time dependant

9.Considering the fluid flow through the thimble as an approximation of fluid through a thin pipe the solutions for the governing equation may be that of a Poiseuille flow as

$V(r) = \Delta P/4\ \mu\ L\ (\ R^2 – r^2)$

μ = viscosity

L,R= Pipe's length & radius

ΔP = pressure drops

r =radial position of considered flow along the cross section

The formula assumes

(a) Incompressible fluid flow

(b) Newtonian fluid flow

(c) time independent / steady flow

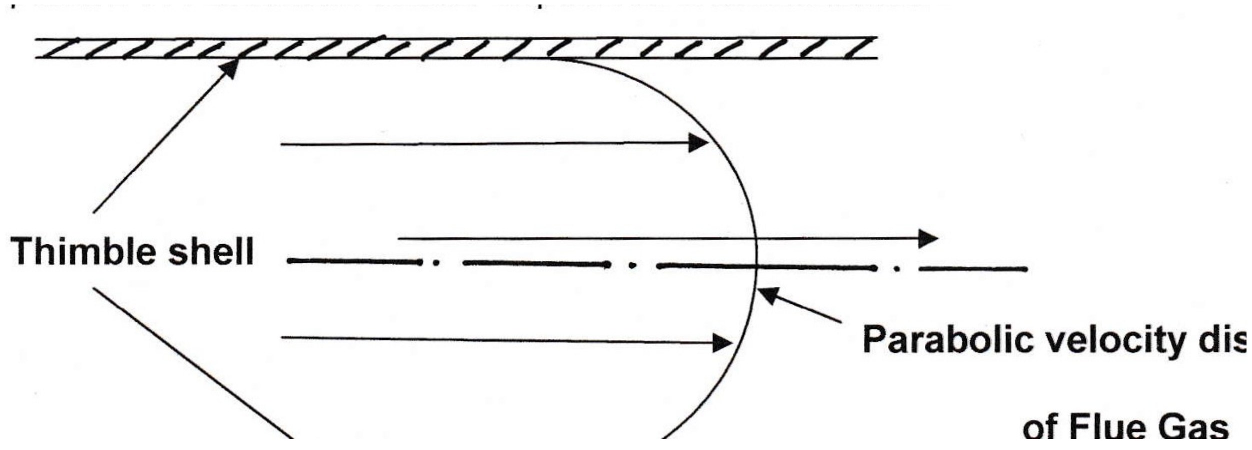
(d) laminar flow

(e) Straight flow

(f) Uniform cross section of flow.

10. The solutions will be modified accordingly in case the flow is Compressible, Unsteady, Non Newtonian, Turbulent and dimensionally variable.

11. Another interesting phenomenon may be possible merging of the boundary layers from two opposing surfaces, thus contributing further to velocity & momentum transfer & hence unique pattern of Particulate Matter deposition characteristics.



The velocity distribution profile (V) of the flue gas flowing through the Thimble may be considered an approximation of the corresponding equation for any laminar, incompressible fluid flow

It reveals a parabolic velocity distribution function as

$V_x = R^2 / 4\mu \, (\Delta P/L) \, (1 - r^2 / R^2)$

$(\Delta P/L)$ = pressure gradient across tube

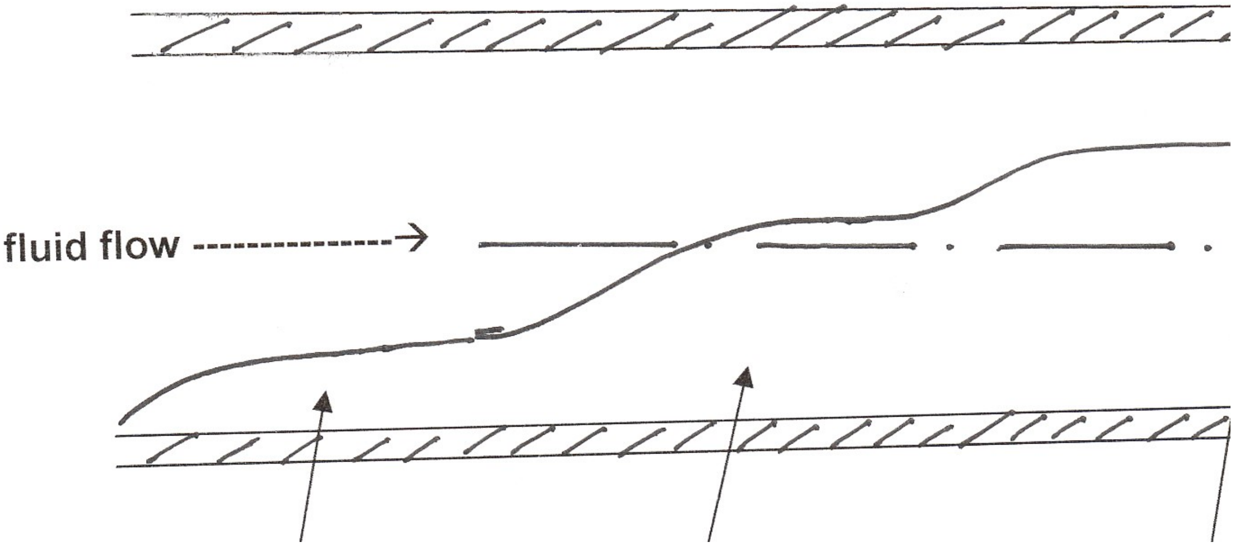
μ = Viscosity

r = radius/distance of arbitrary velocity calculation

R =Radius of the pipe / Thimble

To modify the equation to accommodate a case of compressible flow the incorporation of Compressibility factor $Z = PV_m / RT$ should be considered for further modification of the pressure factor P where V_m = Volume R = Universal gas Constant T = Temperature

If the flow becomes turbulent where the momentum transfer & the boundary layer development becomes different and more intense compared to laminar flow



Laminar Boundary Layer Transitional Boundary Layer Turbulent boundary Layer

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